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Optimal stabilization of partial movements of the frictional speed controller in case of imprecise fulfillment of the conditional connection

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Abstract: This chapter examines the movement of frictional controllers, on which, in addition to passive kinematic connections, an imperfect conditional connection is imposed in the form of a constant angular velocity of the receiving shaft. Differential equations of motion of such speed controllers are obtained, and the question of stability of the controller motion with respect to deviation from the conditional connection is considered. The question of optimal stabilization of the programmed motion of the controller in the vicinity of the manifold determined by the conditional connection is considered. It is shown that the system is controlled by the first approximation. Equations are compiled to determine the coefficients of the Lyapunov function, which solves the problem of optimal stabilization of stationary motions of the speed controller. An explicit form of the control force is obtained, which implements the conditional connection. The question of the influence of the elasticity of the intermediate wheel on the stability of the stationary motion of the regulator is considered and conditions are obtained under which the stability in the first approximation takes place.

Keywords: imperfect connections; combining links; frictional forces; friction regulator; stability and motion stabilization.

Introduction

N.G. Chetaev proposed an algorithm for the release of bonds - the parametric release of material systems. The liberation of the system is called any of its transformation [2], which, without narrowing the variety of admissible states, makes the system more free in each of its states. The system at each moment becomes more free if in this state of it its variety of accelerations, which it can receive in real motion, expands. With such an extension, all states and accelerations allowed for a non-free system are considered acceptable for a liberated system. Let there be a system whose position is determined by the coordinates $x_1, x_2, \ldots, x_n$.

Let compatible connections be imposed on such a system

$$f_\alpha(x_1, x_2, \ldots, x_n) = 0 \quad \alpha = 1, \ldots, a,$$

$$\varphi_\beta(x_1, x_2, \ldots, x_n, \dot{x}_1, \dot{x}_2, \ldots \dot{x}_n, t) = 0 \quad \beta = 1, 2, \ldots, b.$$

If we introduce generalized coordinates $q_1, q_2, \ldots, q_k \, (k = n - a)$, then the variety of admissible states can be represented as

$$x_i = a_i(q_1, q_2, \ldots, q_k),$$

$$\dot{x}_i = b_i(q_1, q_2, \ldots, q_k, p_1, p_2, \ldots p_s, t) \quad s = n - a - b, \tag{1}$$

Where $p_1, p_2, \ldots p_s$ - independent speed parameters.

For a parametrically free system, we can write

$$x_i = a_i'(Q_1, Q_2, \ldots, Q_{k'}, t),$$

$$\dot{x}_i = b_i'(Q_1, Q_2, \ldots, Q_k, P_1, P_2, \ldots P_{s'}, t) \; ,$$

Where $Q_k -$ Lagrangian coordinates, $P_S -$ independent speed parameters.

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Moreover, according to the definition $k' > k,\ \ s' > s$, since for a released system all states of an unreleased system must be admissible. Therefore, for any system of values $q_1', q_2', \ldots, q_k'$, $p_1', p_2', \ldots p_s'$ at any moment there is a system of values $Q_1', Q_2', \ldots, Q_{k'}'$, $P_1', P_2', \ldots P_{s'}'$, for which the equalities

$$a_i(q_1', q_2', \ldots, q_k') = a_i'(Q_1', Q_2', \ldots, Q_{k'}', t),$$

$$b_i(q_1', q_2', \ldots, q_k', p_1', p_2', \ldots p_s', t) = b_i'(Q_1', Q_2', \ldots, Q_k', P_1', P_2', \ldots P_{s'}', t) \ .$$

Following the release algorithm outlined above, for the conditional connection we will have

$$\omega_2 = \omega_2^0 + \zeta, \tag{2}$$

$$\dot{\zeta} = \xi,$$

Where ζ - a parameter representing the deviation from the conditional link.

Accordingly, passive links are also transformed, and will have the form

$$R \cdot \omega_1 = \rho(\omega_2^0 + \zeta),\ r\omega = \rho(\omega_2^0 + \zeta),$$

which corresponds to the definition of parametric release.

To compose the differential equations of the controller motion, we will use the equations in the Appel form.

Regulator acceleration energy $S = \frac{1}{2}(J_1\dot{\omega}_1^2 + J\dot{\omega}^2 + m\ddot{\rho}^2 + J_2\dot{\omega}_2^2) + ..,$ c

taking into account the constraint equations has the following form

$$S' = \frac{1}{2}\left(\frac{J_1}{R^2} + \frac{J}{r^2}\right)(\dot{\zeta}\rho + (\zeta + \omega_2^0)\dot{\rho})^2 + \frac{1}{2}m\ddot{\rho}^2 + \frac{1}{2}J_2\dot{\zeta}^2). \tag{3}$$

Generalized forces corresponding to variables ζ, ρ, will be next

$$\delta A_\rho = -(F_{11}+F_{22})\delta\rho + (u+u_1)\delta\rho = (-(F_{11}+F_{22})+u+u_1)\delta\rho = Q'_\rho\delta\rho\,,$$

$$\delta A_\zeta = M_2\delta\varphi_2 + M_1\delta\varphi_1 = (M_2 + \frac{\rho}{R}M_1)\delta\xi = Q'_\xi\delta\xi\,.$$

Thus, the Appel equations for this problem

$$\frac{\partial S'}{\partial\dot\zeta} = Q'_\xi,$$
$$\frac{\partial S'}{\partial\ddot\rho} = Q'_\rho,$$

can be written explicitly as follows

$$\left(n\rho_0^2 + J_2\right)\dot\zeta + n\left(\zeta+\omega_2^0\right)\dot\rho\rho = M_2 + \frac{\rho}{R}\cdot M_1\,,$$

$$m\ddot\rho = F_1 + F_2 + u + u_1, \qquad (4)$$

Where $n = \frac{J_1}{R^2} + \frac{J}{r^2}$, u_1 - deviation of the control force from the programmed value.

Considering that $\rho = \rho_0 + \rho'$, we have

$$(n\rho^2 + J_2)\dot\zeta + n(\zeta+\omega_2^0)(\rho_0+\rho')\dot\rho = M_2 + \frac{\rho}{R}M_1\,, \qquad (5)$$

$$m\ddot\rho = -(F_{11}+F_{22}) + u + u_1\,.$$

The system of equations (5) represents the equations of a parametrically liberated system in the vicinity of the manifold determined by the conditional constraint, taking into account the imprecise fulfillment of the conditional constraint.

Let us consider the question of optimal stabilization of the programmed motion of the reducer in the vicinity of the manifold determined by the conditional constraint. As is known [5], such a

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problem is reduced to the definition of the Lyapunov function (if the system is controllable) in the form of a quadratic form with an infinite small upper limit.

Since the equations of motion (5) are a non-autonomous system, the Lyapunov function will also depend on time.

Expanding the right-hand sides of the system of equations (5) in the neighborhood $\rho=\rho(t),\quad \zeta=0$ in a series we obtain the equations of the first approximation

$$(n\rho_0^2+J_3)\dot{\zeta}+n\dot{\rho}_0\rho_0\zeta+n\rho_0\omega_2^0\dot{\rho}'+n\omega_2^0\dot{\rho}_0\rho'=\frac{M_1}{R}\rho', \tag{6}$$

$$m\ddot{\rho}'=u_1.$$

In normal form, these equations take the form

$$(n\rho_0^2+J_3)\dot{\zeta}=-n\dot{\rho}_0\rho_0\zeta-n\rho_0\omega_2^0 z+(\frac{M_1}{R}-n\omega_2^0\dot{\rho}_0)\rho', \tag{7}$$

$$\dot{\rho}'=z,$$
$$m\dot{z}=u_1.$$

In the case under consideration, an important role is played by the matrix W, constructed as follows [3, 5]

$$W(t)=\{L_1(t),\ldots,L_n(t)\}. \tag{8}$$

Here $L_k(t)$ - matrices defined by recurrence relations

$$L_1(t)=\begin{pmatrix}0\\0\\1\end{pmatrix},\ \ldots,\ L_{i+1}(t)=\frac{dL_i(t)}{dt}-P(t)\cdot L_i(t)\ . \tag{9}$$

For matrix components W we have

$$\dot{L}_1(t) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$L_2(t) = -\begin{pmatrix} -n \cdot \rho_0 \cdot \dot{\rho}_0 & S_1 & -n \cdot \omega_2^0 \cdot \rho_0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} n \cdot \omega_2^0 \cdot \rho_0 \\ -1 \\ 0 \end{pmatrix}$$

$$L_2(t) = \begin{pmatrix} n \cdot \omega_2^0 \cdot \dot{\rho}_0 \\ 0 \\ 0 \end{pmatrix}, \; L_3(t) = \dot{L}_2(t) - P \cdot L_2(t),$$

$$P \cdot L_2 = \begin{pmatrix} -n \cdot \rho_0 \cdot \dot{\rho}_0 & S_1 & -n \cdot \omega_2^0 \cdot \rho_0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} n \cdot \omega_2^0 \cdot \rho_0 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} -n^2 \cdot \rho_0^2 \cdot \omega_2^0 \cdot \dot{\rho}_0 - S_1 \\ 0 \\ 0 \end{pmatrix},$$

$$L_3(t) = \begin{pmatrix} n \cdot \omega_2^0 \cdot \dot{\rho}_0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} n^2 \cdot \rho_0^2 \cdot \omega_2^0 \cdot \dot{\rho}_0 + S_1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \dot{\rho}_0 \cdot n \cdot \omega_2^0 (1 + n \cdot \rho_0^2) + S_1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{\mu_1}{r} + n^2 \cdot \omega_2^0 \cdot \rho_0^2 \cdot \dot{\rho}_0 \\ 0 \\ 0 \end{pmatrix}$$

$$W = \begin{pmatrix} 0 & n \cdot \omega_2^0 \cdot \dot{\rho}_0 & \frac{\mu_1}{r} \cdot n^2 \cdot \omega_2^0 \cdot \rho_0^2 \cdot \dot{\rho}_0 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}. \qquad (10)$$

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Matrix rank W is equal to the order of the system, since $w = \rho_0{}^2 \dot{\rho}_0 \omega_2^0 \neq 0$. This means that the system is controllable [5] in the first approximation. Therefore, this problem has a solution.

As an optimality criterion, we take the integral

$$I = \int_0^\infty (\zeta^2 + \rho'^2 + z^2 + u_1^2) dt . \tag{11}$$

Then, equations in normal form (6) will take the following form:

$$\begin{cases} \dot{\xi} = \dfrac{1}{(n\rho_c^2 + J^2)} (-n\rho_0 \cdot n\dot{\rho}_0 \xi - n\omega_2^0 \cdot \rho_0 z + S_1 \rho^{'}), \\ \dot{\rho}' = z, \\ m\dot{z} = u_1. \end{cases} \tag{12}$$

$$n_1 = \frac{n}{\omega \cdot \rho_0^2 + J^2}, \qquad S'_1 = \frac{\dfrac{M_1}{R} - n\omega_2^0 \dot{\rho}_0}{n\rho_0^2 + J_2}.$$

To solve the problem of optimal stabilization, it is necessary to construct the Lyapunov function [5]. According to the theory, the Lyapunov function will be sought in the form:

$$V = C_{11}\xi^2 + C_{22} \cdot z^2 + C_{33}\rho'^2 + 2C_{12} \cdot \xi \cdot z + C_{23}\rho' z . \tag{13}$$

Let us compose the Bellman function [47]

$$B[V, t, \xi, z, \rho', u] = \dot{C}_{11}\xi^2 + \dot{C}_{22}z^2 + C_{33}\rho'^2 + 2C_{12}\xi \cdot z + 2C_{13}\xi z + \dot{C}_{23}\rho' z +$$

$$+ (2C_{11}\xi + 2C_{12}\rho' + 2C_{13}z) \cdot (-n_1\rho\rho_0\xi - n_1\omega_2^0 \cdot \rho z + S'\rho') + (2C_{22}z) + 2C_{13}\xi + 2C_{23}\rho') \cdot$$

$$\cdot U_1 + (2C_{33}\rho' + 2C_{12}\xi + 2C_{23}z) \cdot z + \xi^2 + z^2 + \rho'^2 + u_1^2$$

$$\frac{dB}{dU_1} = 2u_1 + 2C_{22}z + 2C_{13}\xi + 2C_{23}\rho' = 0.$$

Next, we compose equations for determining the coefficients of the Lyapunov function

$$\begin{cases} \dot{C}_{11} = 2 \cdot C_{11} \cdot n_1 \cdot \rho \cdot \dot{\rho}_0 + C_{13}^2 - 1, \\ C_{22} = 2 \cdot C_{13} \cdot n_1 \cdot \omega_2^0 \cdot \rho_0 + C_{22}^2 - 2 \cdot C_{23} - 1, \\ \dot{C}_{33} = -2 \cdot C_{12} \cdot S' + C_{23}^2 - 1, \\ \dot{C}_{13} = u_1 \cdot \omega_2^0 \cdot \rho_0 \cdot C_{11} + C_{13} \cdot u_1 \cdot \rho_0 \cdot \dot{\rho}_0 + 2 \cdot C_{13} \cdot C_{22} - 2 \cdot C_{12}, \\ \dot{C}_{12} = -C_{11} \cdot S' + C_{12} \cdot u_1 \cdot \rho_0 \cdot \dot{\rho}_0 + 2 \cdot C_{13} \cdot C_{23}, \\ \dot{C}_{23} = u_1 \cdot \omega_2^0 \cdot \rho_0 \cdot C_{12} - S' \cdot C_{13} + 2 \cdot C_{22} \cdot C_{23}, \end{cases} \tag{14}$$

Where $n_1 = \dfrac{n}{n\rho_0^2 + J_2}$, with initial conditions $c_{ij}(0) = 0$.

Seeking optimal control u_1 is found by the formula

$$u_1 = -(c_{33}z + c_{13}\zeta + c_{23}\rho'). \tag{15}$$

Thus, if it is possible to find a particular solution of Eq. (5), then the problem of stabilization of motions in the neighborhood of states admitted by the conditional constraint is solved by the above algorithm.

The last chapter explores the movements of the three types of friction speed controllers. In contrast to works [4, 1], in addition to passive kinematic connections, which are realized with the help of Coulomb friction forces, a conditional connection is imposed on the regulators in the form of a constant angular velocity of the receiving shaft.

With the help of the axiom of release from constraints, an equation was obtained to determine the law of motion of the intermediate shaft, as well as an explicit form of the control force. The resulting equation for the intermediate shaft in the case of the regulator shown in Figure 3.2., In the

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general case, is a nonlinear non-autonomous first-order equation and is not integrated in the general case.

Despite this, in the following special cases, a solution for the intermediate shaft equation was obtained:

The angular velocity of the receiving shaft is sufficiently high, i.e., as a small parameter, we can take $\varepsilon = \frac{1}{\omega_2^0}$ and the solution can be constructed using the Poincaré small parameter method.

The torque applied to the take-up shaft is constant.

The question of the influence of the elasticity of the intermediate wheel on the stability of the steady motion of the speed controller is also considered. Relationships are obtained, under which the stability of the particular motion of the controller takes place. In this case, elasticity is taken into account using a discrete rolling model (model of M.V. Keldysh). Following the algorithm for freeing from conditional constraints, the issue of optimal stabilization of the programmed motion of the speed controller in the vicinity of the manifold determined by the conditional constraint is considered. Analytical calculations have shown that the system is controllable, and it is possible to construct a Lyapunov function that solves the problem of optimal stabilization.

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