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THE THIRD BOUNDARY VALUE PROBLEM FOR A FIFTH ORDER EQUATION WITH MULTIPLE CHARACTERISTICS IN A FINITE DOMAIN

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Abstract: This article describes the third boundary value problem for a fifth order equation with many characteristics in the number domain.

Keywords: functions, equations, integration, interval, equation.

I. Formulation of the problem

In area $D=\{(x,y):0<x,y<1\}$ consider the equation

$$\frac{\partial^5 u}{\partial x^5}+\frac{\partial^2 u}{\partial y^2}=0. \qquad (1)$$

We say $u(x,y)$ that a regular solution to equation (1) if it satisfies equation (1) in the domain and belongs D to the class $C_{x,y}^{5,2}(D)\,I\ \ C_{x,y}^{4,1}(\bar{D})$.

A task A. Find a regular solution to equation (1) in the domain D satisfying boundary conditions

$$\begin{cases}\alpha u(x,0)+\beta u_y(x,0)=0,\\ \gamma u(x,1)+\delta u_y(x,1)=0,\end{cases} \qquad (2)$$

$$\begin{cases}u(0,y)=\varphi_1(y),\ \ u_x(0,y)=\varphi_2(y),\\ u(1,y)=\varphi_3(y),\ u_x(1,y)=\varphi_4(y),\ \ u_{xx}(1,y)=\varphi_5(y)\end{cases} \qquad (3)$$

Here $\alpha,\beta,\gamma,\delta$ - constant numbers, and $\alpha^2+\beta^2\neq 0,\ \gamma^2+\delta^2\neq 0.$

Where $\varphi_i(y)\in C^4[0,1]$, $\alpha\varphi_i(0)+\beta\varphi_i'(0)=0,\ \alpha\varphi_i''(0)+\beta\varphi_i'''(0)=0,$ $\gamma\varphi_i(1)+\delta\varphi_i'(1)=0,\quad \gamma\varphi_i''(1)+\delta\varphi_i'''(1)=0,\ \ i=\overline{1,5}.$

Note that the problem for a third-order equation was studied in [1]. In [2, 3], various boundary value problems in semi-infinite domains were studied for the fifth equation. In [4, 5], the first and second boundary value problems were studied for a fifth-order equation. In this paper, the third boundary value problem is investigated in the quadratic domain.

II. Uniqueness of the solution

Theorem 1 If the problem has a solution and $\alpha\beta\le 0,\ \delta\gamma\ge 0,$ then it is unique.

Evidence. Suppose the opposite, let $u_1(x,y)$ and $u_2(x,y)$ are solutions to the problem A. Then $u(x,y)=u_1(x,y)-u_2(x,y)$ is a solution to a homogeneous problem A. In area D consider the identity

$$\frac{\partial}{\partial x}(uu_{xxxx})-\frac{\partial}{\partial x}(u_xu_{xxx})+\frac{1}{2}\frac{\partial}{\partial x}(u_{xx}^2)+\frac{\partial}{\partial y}(uu_y)-u_y^2=0$$

integrating this identity over the region D, we have

$$\int_0^1 u(1,y)u_{xxxx}(1,y)dy-\int_0^1 u(0,y)u_{xxxx}(0,y)dy-\int_0^1 u_x(1,y)u_{xxx}(1,y)dy+$$
$$+\int_0^1 u_x(0,y)u_{xxx}(0,y)dy+\frac{1}{2}\int_0^1 u_{xx}^2(1,y)dy-\frac{1}{2}\int_0^1 u_{xx}^2(0,y)dy+$$
$$+\int_0^1 u(x,1)u_y(x,1)dx-\int_0^1 u(x,0)u_y(x,0)dx-\iint_D u_y^2(x,y)dxdy=0.$$

Taking into account the homogeneous boundary conditions of the problem A, demanding $\alpha\ne 0,\ \gamma\ne 0$ get

$$\frac{1}{2}\int_0^1 u_{xx}^2(0,y)dy+\frac{\delta}{\gamma}\int_0^1 u_y^2(x,1)dx-\frac{\beta}{\alpha}\int_0^1 u_y^2(x,0)dx+\iint_D u_y^2(x,y)dxdy=0,$$

hence, by the hypothesis of the theorem 1 $u_y(x,y)=0$, then $u(x,y)=f(x)$. Given the condition $u(x,0)=0$ we get that $f(x)\equiv 0$ or $u(x,y)=0$. In cases $\beta\ne 0,\ \delta\ne 0;\ \alpha\ne 0,\delta\ne 0;\ \gamma\ne 0,\beta\ne 0$ similarly, we obtain the equality $u(x,y)\equiv 0$ in $\overline{D}$. Theorem 1 is proved.

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III. Existence of a solution

The solution to the problem will be sought by the Fourier method

$$u(x,y)=X(x)Y(y) \quad (4)$$

Putting (4) into (1), we get

$$X^{(5)}-\lambda^5 X=0, \quad (5)$$

$$Y''+\lambda^5 Y=0, \quad (6)$$

from (6) and (2) we will have

$$\begin{cases} Y''+\lambda^5 Y=0, \\ \alpha Y(0)+\beta Y'(0)=0, \\ \gamma Y(1)+\delta Y'(1)=0. \end{cases} \quad (7)$$

Proceeding as in [1], to find the eigenvalues λ^5 we obtain the transcendental equation

$$ctg\sqrt{\lambda^5}=\frac{\alpha\gamma+\delta\beta\lambda^5}{\sqrt{\lambda^5}(\gamma\beta-\alpha\delta)},$$

whence it follows that $\sqrt{\lambda_n^5}=\pi n+\varepsilon_n$, Where $\lim\limits_{n\to\infty}\varepsilon_n=0$, or $\lambda_n^5=O(n^2)$, $n\to\infty$.

The corresponding eigenfunctions are [1]

$$Y_n(y)=\left(\alpha\sin\sqrt{\lambda_n^5}y-\beta\sqrt{\lambda_n^5}\cos\sqrt{\lambda_n^5}y\right)A_n, \quad (8)$$

Where A_n - arbitrary constants.

The orthogonality of the system of functions (8) is proved as in the work [1]. Take two arbitrary functions $Y_n(y)$ and $Y_m(y)$.
Then

$$Y_n''+\lambda_n^5 Y_n=0, \qquad Y_m''+\lambda_m^5 Y_m=0.$$

Multiplying the first identity by Y_m, and the second - on Y_n, then subtracting the obtained equalities term by term, we find

$$Y_m Y_n''+\lambda_n^5 Y_n Y_m-Y_n Y_m''-\lambda_m^5 Y_n Y_m=\left(\lambda_n^5-\lambda_m^5\right)Y_n Y_m+Y_m Y_n''-Y_n Y_m''=0,$$

or

$$\left(\lambda_n^5-\lambda_m^5\right)Y_nY_m=\frac{d}{dy}\left(Y_mY_n'-Y_nY_m'\right).$$

Integrating this identity in the interval and taking into account the conditions of problem (7), we obtain at $\alpha\neq 0,\ \gamma\neq 0$:

$$\left(\lambda_n^5-\lambda_m^5\right)\int_0^1 Y_nY_m dy=0.$$

As $m\neq n,\ \ \lambda_n^5-\lambda_m^5\neq 0,$ hence

$$\int_0^1 Y_n(y)Y_m(y)dy=0.$$

Calculating the norm of the eigenfunctions $Y_n(y)$ в $L_2[0,1],$ we have

$$\left\|Y_n(y)\right\|^2=A_n^2\int_0^1 Y_n^2(y)dy=$$

$$=A_n^2\left[\frac{1}{2}\left(\alpha^2+\beta^2\lambda_n^5-\alpha\beta\right)+\left(\frac{\beta^2\sqrt{\lambda_n^5}}{4}-\frac{\alpha^2}{4\sqrt{\lambda_n^5}}\right)\sin 2\sqrt{\lambda_n^5}+\frac{\alpha\beta}{2}\cos 2\sqrt{\lambda_n^5}\right].$$

Then

$$Y_n(y)=\frac{1}{\left\|Y_n\right\|^2}\left(\alpha\sin\sqrt{\lambda_n^5}y-\beta\sqrt{\lambda_n^5}\cos\sqrt{\lambda_n^5}y\right).$$

Let's evaluate the expression $\dfrac{\left|Y_n(y)\right|}{\left\|Y_n(y)\right\|}$. As

$$\left|Y_n(y)\right|\leq\left|\alpha\sin\sqrt{\lambda_n^5}y-\beta\sqrt{\lambda_n^5}\cos\sqrt{\lambda_n^5}y\right|\leq|\alpha|+|\beta|\sqrt{\lambda_n^5},$$

get

$$\left(\frac{\left|Y_n(y)\right|}{\left\|Y_n\right\|}\right)^2\leq\frac{\left(|\alpha|+|\beta|\sqrt{\lambda_n^5}\right)^2}{\left\|Y_n\right\|^2}=$$

$$=\frac{\alpha^2+2|\alpha||\beta|\sqrt{\lambda_n^5}+\beta^2\lambda_n^5}{\frac{1}{2}\left(\alpha^2+\beta^2\lambda_n^5-\alpha\beta\right)+\left(\frac{\beta^2\sqrt{\lambda_n^5}}{4}-\frac{\alpha^2}{4\sqrt{\lambda_n^5}}\right)\sin 2\sqrt{\lambda_n^5}+\frac{\alpha\beta}{2}\cos 2\sqrt{\lambda_n^5}}\to 2,$$

at $n\to\infty$. From this we conclude that starting from some number n

the inequality is true

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$$\left(\frac{|Y_n(y)|}{\|Y_n\|}\right)^2 < B \text{, где } B \geq 2.$$

In what follows, we will assume that the system of eigenfunctions is orthonormal; the boundedness of the eigenfunction was shown above.

The general solution to equation (5) has the form

$$X(x) = C_1 e^{\lambda x} + e^{\lambda\alpha_2 x}(C_2 \cos\lambda\beta_2 x + C_3 \sin\lambda\beta_2 x) + e^{-\lambda\alpha_1 x}(C_4 \cos\lambda\beta_1 x + C_5 \sin\lambda\beta_1 x), \quad (9)$$

Where

$$\alpha_1 = \cos\theta_1,\ \beta_1 = \sin\theta_1,\ \alpha_2 = \cos\theta_2,\ \beta_2 = \sin\theta_2,\ \theta_1 = \frac{\pi}{5},\ \theta_2 = \frac{2\pi}{5},$$

$C_i - (i = \overline{1,5})$ - arbitrary constants.

Due to the linearity and homogeneity of equation (1), any sum will also be a solution

$$u(x,y) = \sum_{n=1}^{\infty} X_n(x) Y_n(y). \quad (10)$$

The function defined by series (10) satisfies conditions (2), since all terms of the series satisfy them. Satisfying conditions (3), we obtain the system of equations

$$\begin{cases} C_{1n} + C_{2n} + C_{4n} = A_{1n}, \\ C_{1n} + \cos\theta_2 C_{2n} + \sin\theta_2 C_{3n} - \cos\theta_1 C_{4n} + \sin\theta_1 C_{5n} = \dfrac{A_{2n}}{\lambda_n}, \\ C_{1n}e^{\lambda_n} + C_{2n}e^{\lambda_n\alpha_2}\cos\lambda_n\beta_2 + C_{3n}e^{\lambda_n\alpha_2}\sin\lambda_n\beta_2 + C_{4n}e^{-\lambda_n\alpha_1}\cos\lambda_n\beta_1 + \\ \qquad + C_{5n}e^{-\lambda_n\alpha_1}\sin\lambda_n\beta_1 = A_{3n}, \\ C_{1n}e^{\lambda_n} + C_{2n}e^{\lambda_n\alpha_2}\cos(\lambda_n\beta_2 + \theta_2) + C_{3n}e^{\lambda_n\alpha_2}\sin(\lambda_n\beta_2 + \theta_2) - \\ \qquad - C_{4n}e^{-\lambda_n\alpha_1}\cos(\lambda_n\beta_1 - \theta_1) - C_{5n}e^{-\lambda_n\alpha_1}\sin(\lambda_n\beta_1 - \theta_1) = \dfrac{A_{4n}}{\lambda_n}, \\ C_{1n}e^{\lambda_n} + C_{2n}e^{\lambda_n\alpha_2}\cos(\lambda_n\beta_2 + 2\theta_2) + C_{3n}e^{\lambda_n\alpha_2}\sin(\lambda_n\beta_2 + 2\theta_2) + \\ \qquad + C_{4n}e^{-\lambda_n\alpha_1}\cos(\lambda_n\beta_1 - 2\theta_1) + C_{5n}e^{-\lambda_n\alpha_1}\sin(\lambda_n\beta_1 - 2\theta_1) = \dfrac{A_{5n}}{\lambda_n^2}, \end{cases} \quad (11)$$

Where

$$A_{in} = 2\int_0^1 \varphi_i(y)Y_n(y)dy,\ i=\overline{1,5}. \quad (12)$$

Solving system (11), we get

$$C_{in} = \frac{\Delta_i}{\Delta}, \quad i=\overline{1,5}.$$

Let us show that $\Delta \neq 0$. For this, we prove the following lemma.

Lemma: Boundary problem

$$\begin{cases} X^{(5)} - \lambda^5 X = 0, \\ X(0) = X'(0) = X(1) = X'(1) = X''(1) = 0, \end{cases}$$

has only a trivial solution.

Evidence: Suppose the opposite, let $X(x) \neq 0$. Consider the identity

$$X\left(X^{(5)} - \lambda^5 X\right) = 0,$$

or

$$\left(XX^{(4)} - X'X''' + \frac{1}{2}(X'')^2\right)' - \lambda^5 X^2 = 0,$$

integrating over the area $(0 < x < 1)$, we have

$$\int_0^1 \left(XX^{(4)} - X'X''' + \frac{1}{2}(X'')^2\right)' dx - \lambda^5 \int_0^1 X^2 dx = 0,$$

$$\left(XX^{(4)} - X'X''' + \frac{1}{2}(X'')^2\right)\Bigg|_0^1 - \lambda^5 \int_0^1 X^2 dx = 0,$$

$$X(1)X^{(4)}(1) - X(0)X^{(4)}(0) - X'(1)X'''(1) + X'(0)X'''(0) + \frac{1}{2}\left(X''(1)\right)^2 -$$

$$-\frac{1}{2}\left(X''(0)\right)^2 - \lambda^5 \int_0^1 X^2 dx = 0,$$

taking into account the boundary conditions, we obtain

$$\frac{1}{2}\left(X''(0)\right)^2 + \lambda^5 \int_0^1 X^2 dx = 0,$$

as $\lambda > 0$, then $X(x) \equiv 0$.

Hence, the system of equations (11) has a unique solution. The determinant of the system is:

$$\Delta = \begin{vmatrix} A_{2\times3} & B_{2\times2} \\ C_{3\times3} & D_{3\times2} \end{vmatrix},$$

Where

$$A_{2\times3} = \begin{vmatrix} 1 & 1 & 0 \\ 1 & \cos\theta_2 & \sin\theta_2 \end{vmatrix}, \quad B_{2\times2} = \begin{vmatrix} 1 & 0 \\ -\cos\theta_1 & \sin\theta_1 \end{vmatrix},$$

$$C_{3\times3} = \begin{vmatrix} e^{\lambda_n} & e^{\lambda_n\alpha_2}\cos\lambda_n\beta_2 & e^{\lambda_n\alpha_2}\sin\lambda_n\beta_2 \\ e^{\lambda_n} & e^{\lambda_n\alpha_2}\cos(\lambda_n\beta_2+\theta_2) & e^{\lambda_n\alpha_2}\sin(\lambda_n\beta_2+\theta_2) \\ e^{\lambda_n} & e^{\lambda_n\alpha_2}\cos(\lambda_n\beta_2+2\theta_2) & e^{\lambda_n\alpha_2}\sin(\lambda_n\beta_2+2\theta_2) \end{vmatrix},$$

$$D_{3\times2} = \begin{vmatrix} e^{-\lambda_n\alpha_1}\cos\lambda_n\beta_1 & e^{-\lambda_n\alpha_1}\sin\lambda_n\beta_1 \\ -e^{-\lambda_n\alpha_1}\cos(\lambda_n\beta_1-\theta_1) & -e^{-\lambda_n\alpha_1}\sin(\lambda_n\beta_1-\theta_1) \\ e^{-\lambda_n\alpha_1}\cos(\lambda_n\beta_1-2\theta_1) & e^{-\lambda_n\alpha_1}\sin(\lambda_n\beta_1-2\theta_1) \end{vmatrix}.$$

Find the largest exponential power that comes in when calculating the determinant Δ. Since in the determinant $C_{3\times3}$ all exponents have positive degrees, then obviously the largest exponent degree is obtained by calculating the product of the following determinants

$$|C_{3\times3}| \cdot |B_{2\times2}|.$$

Let's calculate each determinant separately.

$$B = \begin{vmatrix} 1 & 0 \\ -\cos\theta_1 & \sin\theta_1 \end{vmatrix} = \sin\theta_1,$$

$$C = \begin{vmatrix} e^{\lambda_n} & e^{\lambda_n\alpha_2}\cos\lambda_n\beta_2 & e^{\lambda_n\alpha_2}\sin\lambda_n\beta_2 \\ e^{\lambda_n} & e^{\lambda_n\alpha_2}\cos(\lambda_n\beta_2+\theta_2) & e^{\lambda_n\alpha_2}\sin(\lambda_n\beta_2+\theta_2) \\ e^{\lambda_n} & e^{\lambda_n\alpha_2}\cos(\lambda_n\beta_2+2\theta_2) & e^{\lambda_n\alpha_2}\sin(\lambda_n\beta_2+2\theta_2) \end{vmatrix} = 4e^{\lambda_n+2\lambda_n\alpha_2}\sin\theta_2\sin^2\frac{\theta_2}{2}.$$

From here

$$\Delta = e^{\lambda_n+2\lambda_n\alpha_2}K + f(\lambda_n),$$

Where

$$K = 4\sin\theta_1\sin\theta_2\sin^2\frac{\theta_2}{2},$$

$$f(\lambda_n) = o\left(e^{\lambda_n+2\lambda_n\alpha_2}\right) \quad \text{at} \quad \lambda_n \to +\infty.$$

Let's estimate Δ

$$|\Delta| = e^{\lambda_n+2\lambda_n\alpha_2}\left|K + e^{-(\lambda_n+2\lambda_n\alpha_2)}f(\lambda_n)\right|,$$

as

$$\lim_{\lambda_n\to\infty} e^{-(\lambda_n+2\lambda_n\alpha_2)} f(\lambda_n)=0,$$

then

$$\forall\varepsilon>0,\ \varepsilon<K \quad \exists N_1 \big|\ \forall n>N_1 \Rightarrow \left|e^{-(\lambda_n+2\lambda_n\alpha_2)} f(\lambda_n)\right|<\varepsilon,$$

hence at $n>N_1$ inequality holds

$$\left|K+e^{-(\lambda_n+2\lambda_n\alpha_2)} f(\lambda_n)\right|>K-\left|e^{-(\lambda_n+2\lambda_n\alpha_2)} f(\lambda_n)\right|>K-\varepsilon,$$

denote

$$M_1=\min_{n=\overline{1,N_1}}\left|K+e^{-(\lambda_n+2\lambda_n\alpha_2)} f(\lambda_n)\right|,$$

according to lemma $M_1\neq 0$, from here

$$\frac{1}{|\Delta|}\leq\frac{1}{Me^{\lambda_n+2\lambda_n\alpha_2}},$$

Where

$$M=\min\{M_1;K-\varepsilon\}.$$

Now we get estimates for $C_{in},\ \ i=\overline{1,5}$. Calculations show that the following estimates hold for the algebraic complement $|\Delta_i|,\ i=\overline{1,5}$:

$$|\Delta_1|\leq M_1e^{2\lambda_n\alpha_2}\sum_{i=1}^{5}|A_{in}|\ ,\quad |\Delta_2|\leq M_2e^{\lambda_n+\lambda_n\alpha_2}\sum_{i=1}^{5}|A_{in}|,\quad |\Delta_3|\leq M_3e^{\lambda_n+\lambda_n\alpha_2}\sum_{i=1}^{5}|A_{in}|,$$

$$|\Delta_4|\leq M_4e^{\lambda_n+2\lambda_n\alpha_2}\sum_{i=1}^{5}|A_{in}|,\quad |\Delta_5|\leq M_5e^{\lambda_n+2\lambda_n\alpha_2}\sum_{i=1}^{5}|A_{in}|,$$

Where

$$A_{in}=\frac{1}{\lambda_n^{10}}\frac{1}{\|Y_n\|^2}\int_0^1\varphi_i^{(4)}(\eta)Y_n(\eta)\,d\eta=O\left(\frac{1}{n^4}\right),\qquad i=1,2,3.$$

Hence, for the coefficients C_{in} we obtain the following estimates

$$|C_{1n}|=\frac{|\Delta_1|}{|\Delta|}\leq\frac{M_1e^{2\lambda_n\alpha_2}\sum_{i=1}^{5}|A_{in}|}{Me^{\lambda_n+2\lambda_n\alpha_2}}\leq\frac{N_1}{e^{\lambda_n}n^4}\ ,\quad |C_{2n}|=\frac{|\Delta_2|}{|\Delta|}\leq\frac{M_2e^{\lambda_n+\lambda_n\alpha_2}\sum_{i=1}^{5}|A_{in}|}{Me^{\lambda_n+2\lambda_n\alpha_2}}\leq\frac{N_2}{e^{\lambda_n\alpha_2}n^4},$$

$$|C_{3n}|=\frac{|\Delta_3|}{|\Delta|}\leq\frac{M_3e^{\lambda_n+\lambda_n\alpha_2}\sum_{i=1}^{5}|A_{in}|}{Me^{\lambda_n+2\lambda_n\alpha_2}}\leq\frac{N_3}{e^{\lambda_n\alpha_2}n^4},\quad |C_{4n}|=\frac{|\Delta_4|}{|\Delta|}\leq\frac{M_4e^{\lambda_n+2\lambda_n\alpha_2}\sum_{i=1}^{5}|A_{in}|}{Me^{\lambda_n+2\lambda_n\alpha_2}}\leq\frac{N_4}{n^4},$$

$$|C_{5n}|=\frac{|\Delta_5|}{|\Delta|}\leq\frac{M_5e^{\lambda_n+2\lambda_n\alpha_2}\sum_{i=1}^{5}|A_{in}|}{Me^{\lambda_n+2\lambda_n\alpha_2}}\leq\frac{N_4}{n^4}.$$

Let us prove the uniform convergence of series (10) in the domain D.

$$|u(x,y)| \le \sum_{n=1}^{\infty}\left[|C_{1n}||e^{\lambda_n x}| + (|C_{2n}| + |C_{3n}|)|e^{\lambda_n\alpha_2 x}| + (|C_{4n}| + |C_{5n}|)|e^{-\lambda_n\alpha_1 x}|\right]\cdot|Y_n(y)| \le$$

$$\le \sum_{n=1}^{\infty}\left[\frac{N_1}{e^{\lambda_n(1-x)}n^4} + \frac{N_2}{e^{\lambda_n\alpha_2(1-x)}n^4} + \frac{N_3}{e^{\lambda_n\alpha_2(1-x)}n^4} + \frac{N_4}{n^4} + \frac{N_5}{n^4}\right]\cdot|Y_n(y)| \le \sum_{n=1}^{\infty}\frac{N}{n^4}|Y_n(y)| < \infty .$$

The uniform convergence of a series composed of partial derivatives with respect to a variable up to the fifth order inclusive is shown in a similar way.

For $u_{yy}(x,y)$ we have

$$u_{yy}(x,y) = \sum_{n=1}^{\infty}\left[C_{1n}e^{\lambda_n x} + e^{\lambda_n\alpha_2 x}(C_{2n}\cos\lambda_n\beta_2 x + C_{3n}\sin\lambda_n\beta_2 x) + \right.$$

$$\left. + e^{-\lambda_n\alpha_1 x}(C_{4n}\cos\lambda_n\beta_1 x + C_{5n}\sin\lambda_n\beta_1 x)\right]Y_n''(y),$$

Where

$$Y_n''(y) = -\lambda_n^5 Y_n(y).$$

Then

$$u_{yy}(x,y) = \sum_{n=1}^{\infty}\left[C_{1n}e^{\lambda_n x} + e^{\lambda_n\alpha_2 x}(C_{2n}\cos\lambda_n\beta_2 x + C_{3n}\sin\lambda_n\beta_2 x) + \right.$$

$$\left. + e^{-\lambda_n\alpha_1 x}(C_{4n}\cos\lambda_n\beta_1 x + C_{5n}\sin\lambda_n\beta_1 x)\right](-\lambda_n^5)Y_n(y),$$

$$|u_{yy}| \le \sum_{n=1}^{\infty}\left[|C_{1n}|e^{\lambda_n x} + |C_{2n}|e^{\lambda_n\alpha_2 x} + |C_{3n}|e^{\lambda_n\alpha_2 x} + |C_{4n}|e^{-\lambda_n\alpha_1 x} + |C_{5n}|e^{-\lambda_n\alpha_1 x}\right]|\lambda_n^5||Y_n(y)| \le$$

$$\le MB\sum_{n=1}^{\infty}\left[|A_{1n}| + |A_{2n}| + |A_{3n}| + |A_{4n}| + |A_{5n}|\right]\lambda_n^5.$$

Considering that $\lambda_n^5 = O(n^2)$ and from the estimate A_{in}, get

$$|u_{yy}| \le M_4\sum_{n=1}^{\infty}\frac{1}{n^2} < \infty,\ M_4 = const > 0.$$

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Likewise

$$\left|\frac{\partial^5 u}{\partial x^5}\right| = \left|\frac{\partial^2 u}{\partial y^2}\right| < \infty$$

And so, we have proved the following theorem:

Theorem 2. If functions $\varphi_i(y) \in C^4([0,1])$ and $\alpha\varphi_i(0) + \beta\varphi_i'(0) = 0,$ $\alpha\varphi_i''(0) + \beta\varphi_i'''(0) = 0,$ $\gamma\varphi_i(1) + \delta\varphi_i'(1) = 0,$ $\gamma\varphi_i''(1) + \delta\varphi_i'''(1) = 0,$ $i = \overline{1,5}.$ then a solution to the problem exists and are represented as (10).

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